Binomial Random Variables-
<u>LARGE SAMPLE SIZE</u>

What happens when n is large?

distribution of $X \sim$ normal

What is considered a "large enough" n??

check: $n \cdot p$

$n(1-p) \geq 10$

So if the check passes… (both are ≥ 10)

- We can say $X \sim$ normal
- Calculator: use normalcdf(LB, UB, μ, σ)
- Same μ_x & σ_x from form sheet

Example:

It is said that 75% of people pay their credit card bill on time. If we take a sample of 125 adults, what is the chance that over 80 of them paid their bill on time this past month?

$B(125, 0.75)$

$P(x>80) = \text{normalcdf}\left(80, E99, (125\cdot 0.75), \sqrt{(0.75)(0.25)(125)}\right)$

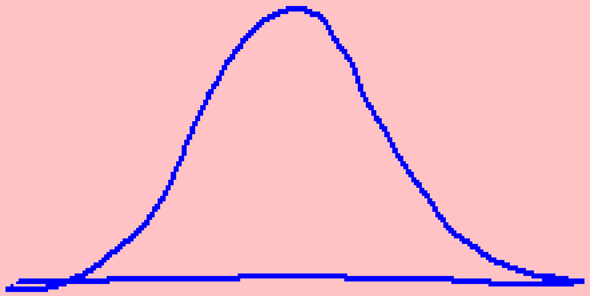
$\boxed{\approx 0.9977}$

check:

$(125)(0.75)$ ✓ ≥ 10

$(125)(0.25)$

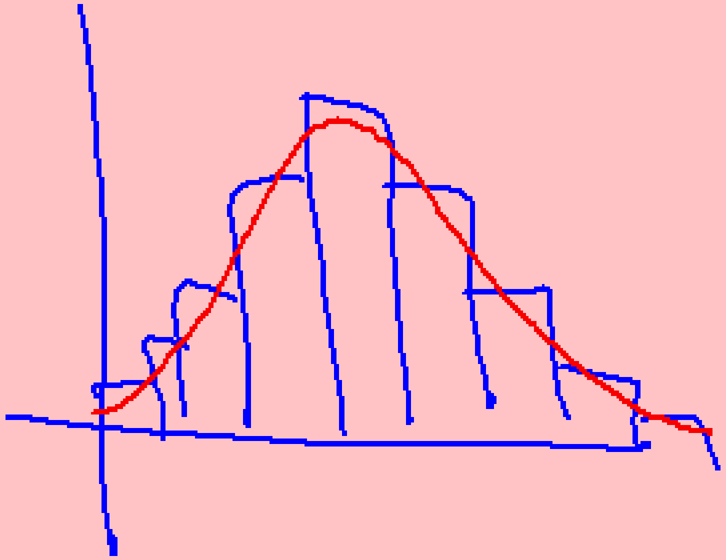
Remember, binomial variables are discrete



But we are approximating it with continuous R.V.

Why?

- easier! normalcdf easier than binomcdf



- prob. basically same

$P(x>80) = 1 - P(x\leq 80) = 0.996$

Sample Proportions

What if we don't want to know about the *NUMBER* of successes? What if we want to know about the *PROPORTION (PERCENT)* of successes? $\cancel{P(X<80)}$ $P(\hat{p}<0.65)$

Keep in mind, we are talking about the same types of problems- still binomial.

p = population proportion

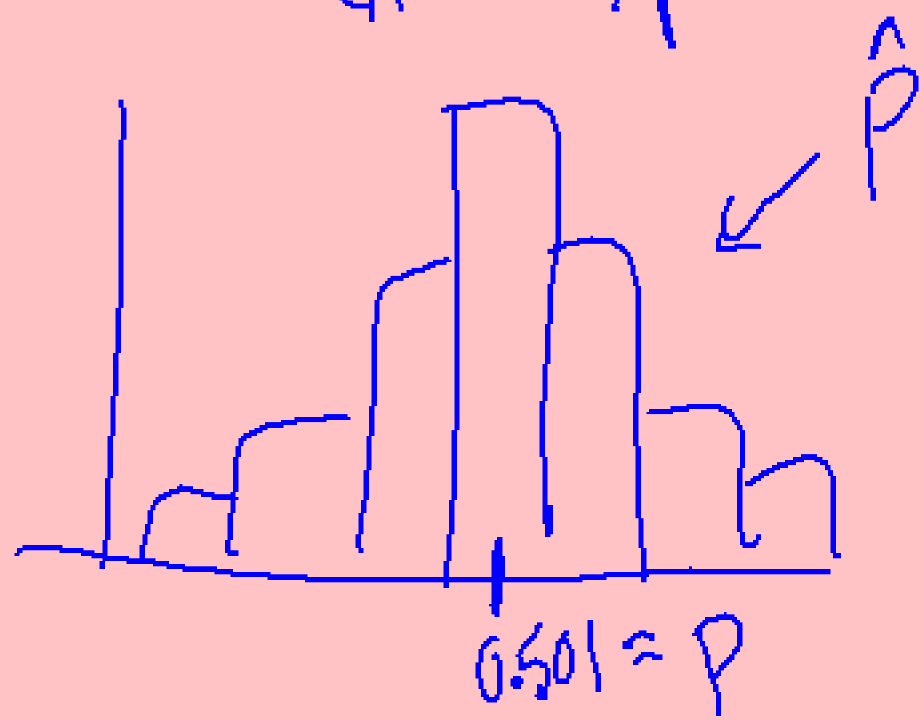
$\hat{p}$ = Sample prop. $= \dfrac{x}{n}$ ← #successes (over) ← total sample size (under)

$\hat{p}$ is continuous. The range is: $0 \le \hat{p} \le 1$

$\hat{p} \sim N\left(p, \sqrt{\dfrac{p(1-p)}{n}}\right)$

$\mu_{\hat{p}}$ (p) $\sigma_{\hat{p}}$ ($\sqrt{\frac{p(1-p)}{n}}$)

dice expts



With sample proportions, since it is approx. normal, we still need to be sure the sample size is big enough.

Check:

$$\left.\begin{matrix} n \cdot p \\ n \cdot (1-p) \end{matrix}\right\} \geq 10$$

If the check passes..... $\hat{p} \sim$ normal
use normalcdf to solve problem

Example:
Assume 15% of adults jog regularly. We survey 1,000 adults what is the probability that more than 14% of our sample jogs?

check

$$\left.\begin{matrix} (1000)(0.15) \\ (1000)(0.85) \end{matrix}\right\} \geq 10 \checkmark$$

$p = 0.15$
$n = 1000$

$$P(\hat{p} > 0.14) = \text{normalcdf}\left(0.14,\ \text{E}99,\ 0.15,\ \sqrt{\frac{(0.15)(0.85)}{1000}}\right)$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.15 \cdot 0.85}{1000}} = 0.0113$$

$$= 0.8121$$

So let's compare...

X	p
• counts (#'s)	• proportions (%)
$P(x \lesseqgtr \underline{\#})$	$P(\hat{p} \gtrless \underline{\quad})$
• discrete r.v.	• continuous r.v.
• $\mu_x = n \cdot p$	• $\mu_{\hat{p}} = p$
$\sigma_x = \sqrt{n \cdot p \cdot (1-p)}$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

Check

$n \cdot p$

$n(1-p) \geq 10$

passes	doesn't pass	passes	doesn't pass
normalcdf	binomcdf pdf	normalcdf	convert prob. into #'s

$n = 200$

$P(\hat{p} > 0.20) = P(X > 40)$

FLOWCHART

binomial

check:
$n \cdot p$
$n(1-p)$ ≥ 10

passes

normalcdf

counts (#'s)
$P(x \gtrless \_\_\_)$
$\mu_x = n \cdot p$
$\sigma_x = \sqrt{n \cdot p(1-p)}$

prop (%)
$P(\hat{p} \gtrless \_\_\_)$
$\mu_{\hat{p}} = p$
$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

doesn't

binomcdf
pdf

X's (counts)
$P(x \gtrless \#)$
$\mu_x = n \cdot p$
$\sigma_x = \sqrt{n \cdot p(1-p)}$

Complete the 5.1 worksheet- Normal Approximations

1 a) $P(x=25)$ b) $P(x \geq 7)$

c) $P(x \leq 230)$ d) $P(15 < X < 20)$

2 a) $15 \geq 10$

15

$\boxed{\text{YES}}$

b) $\cancel{5 \geq 10}$

$\cancel{45}$

$\boxed{\text{NO}}$

c) $\cancel{8} \geq 10$

$\cancel{42}$

$\boxed{\text{NO}}$

3 a) 100

b) 34

c) 50

(4) check

$(700)(0.05)$
$(700)(0.95)$ ≥ 10 ✓

$P(x \geq 50) = \boxed{0.00464}$

$\text{normalcdf}(50, E99, 35, \sqrt{700 \cdot 0.05 \cdot 0.95})$

(5) check

$(400)(0.48)$
$(400)(0.52)$ ≥ 10 ✓

$P(0.45 \leq \hat{p} \leq 0.55) = 0.8826$

$\text{normalcdf}\left(0.45, 0.55, 0.48, \sqrt{\frac{(0.48)(0.52)}{400}}\right)$